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ABSTRACT

This teacher guide is part of the materials prepared for an individualized program for ninth-grade algebra and basic mathematics students. Materials written for the program are to be used with audiovisual lessons recorded on tape cassettes. For an evaluation of the program see ED 086 545. In this guide, the teacher is provided with objectives for each topic area and guided to materials written for a given topic. Three short criterion tests are included for each topic covered. Techniques for factoring polynomials are presented in this package. This work was prepared under an ESEA Title III contract. (JP)

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ED 040 018

ALGEBRA I

Package 7 03-07

SPECIAL PRODUCTS AND FACTORING

Prepared By

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Under a Grant From
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SE 017 571

SPECIAL PRODUCTS AND FACTORING

There are many equations which cannot be solved by using the techniques we have learned to date. For example, to find the answer to a problem which leads to the equation $(x-2)^2 + (2x+5)(2x-5) = -20$, we need more machinery than we now have. After you have mastered "Special Products and Factoring," you will be able to determine that the solution set of the above equation is $\{-1/5, 1\}$.

You will recognize in this unit numerous applications of the distributive property. This is important. If you do not have a good understanding of the right-hand and left-hand distributive properties, you should search out some material on this subject before you begin this package.

Goal of Package 260: Student will be able to solve problems which require the use of special products and factoring in their solution.

Package Objectives:

With 80% accuracy and in less than 45 minutes, you will be able to do the following twice:

1. Given a polynomial, write the polynomial as a product of its greatest monomial factor and another polynomial.

2. Given two binomials which are the sum and difference of two monomials, write their product as a single binomial.

3. Given a binomial that is the difference of two squares, write it as a product of two binomials.

4. Given a binomial, write its square.

5. Given a trinomial square, write it as a binomial squared.

6. Given a factorable quadratic trinomial (not a trinomial square) which has no monomial factor, write it as the product of two binomials.

7. Given a polynomial, write it in factored form if possible by combining several types of factoring.

8. Given a polynomial equation with rational roots, write a step-by-step solution using factoring.

9. Given a problem which leads to a polynomial equation, write a solution to the problem.

I.U. # 03-07-01

Expressing a polynomial as a product
of its greatest monomial factor and
another polynomial.

You will need to recall that:

An *integer* is a member of the set {..., -3, -2, -1, 0, 1,2, 3,...}

A *prime* number is a member of the set {2,3,5,7,11,13,17,19,23,29,31, 37,41,43,47,53,...}
The two most common definitions of a prime number are:

1. a number greater than one whose only integral factors are the number and one.
2. a number that has exactly two positive integral factors.

A *factor* is one of the numbers used in forming a product.

A *product* is the result of the operation of multiplication.

A *monomial* is a constant or a positive integral power of a variable or a product each of whose factors is a constant or a positive integral power of a variable.

A *polynomial* is a monomial or is the indicated sum of two or more monomials.

OBJECTIVES:

1. Given an integer, write it as a product of prime numbers.
2. Given an integer, list all of its integral factors.
3. Given an equation of the form $12a = -60$, write the replacement for a that will result in a true statement.
4. Given a pair of integers, write their greatest common factor.
5. Given an equation of the form $51x^4y^2z^3 = (-17x^3y^2z^2)\ (a)$. write a replacement for a which will result in a true statement.
6. Given an equation of the form $x^2-3x^2y = x^2(a-b)$. find replacements for a and b which will result in a true statement.
7. Given a pair of monomials, write the greatest common monomial that is a factor of both monomials.
8. Given a polynomial, write the polynomial as a product of its greatest monomial factor and another polynomial.

ACTIVITIES:

Study:

S & M: pages 249-251 (Objectives 1, 2, 3, 4, 5)

H, B & W: pages 324-325 (Objectives 1, 2, 3)

S & M: first half of page 253 (Objectives 6, 7, 8)

H, B & W: page 326 (Objectives 6, 8)

Suggested Exercises:

S & M: page 252, ex. 1-8 (Objective 1)

" " , ex. 9-16 (Objective 2)

H, B & W: page 325, ex. 1-21 odds (Objective 3)

S & M: page 252, ex. 17-22 evens (Objective 4)

" " , ex. 29-32 (Objective 5)

H, B & W: pages 326-7, ex. 1-30 assorted (Objective 6)

S & M: page 252, ex. 23-28 (Objective 7)

" page 254, ex. 1-19 odds (Objective 8)

H, B & W: page 327, ex 31-45 odds (Objective 8)

Criterion Test 03-07-01-1

1. Rewrite the following integers as the product of prime numbers.

(a) 85

(b) 72

2. Write all of the positive integral factors of each number.

(a) 18

(b) 19

3. Write the replacement for x which will make each statement true.

(a) $75 = 5x$

(b) $-17x = 17$

4. Write the greatest common factor for each of the following pairs of integers.

(a) 36; 60

(b) 315; 350

5. Write a replacement for a which will result in a true statement in each of the following equations.

(a) $5x^3y^2 = 5xy\ (a)$

(b) $25x^4y^2 = -x^3(a)$

6. Write the replacements for a and b in the following equations which will result in a true statement.

(a) $3x + 6y = 3(a + b)$

(b) $x^2 - 3x^2y = x^2(a + b)$

7. Write the greatest common monomial that is a factor of each of the following pairs of monomials.

(a) $39a^2b;\ 65ab^4$

(b) $30r^2s;\ 45rs$

8. Rewrite each of the following polynomials as the product of its greatest monomial factor and another polynomial.

(a) $5x + 5y$

(b) $ax - bx^2$

(c) $6a^2 + 12ab + 4b^2$

(d) $3a^2b + 9ab^2 + 12ab$

Criterion Test 03-07-01-2

1. Rewrite the following integers as the product of prime numbers.

(a) 85

(b) 36

2. Write all of the positive integral factors of each number.

(a) 18

(b) 21

3. Write the replacement for x which will make each statement true.

(a) $75 = 5x$

(b) $-2x = -10$

4. Write the greatest common factor for each of the following pairs of integers.

(a) 36; 60

(b) 45; 60

5. Write a replacement for a which will result in a true statement in each of the following equations.

(a) $5x^3y^2 = 5xy(a)$

(b) $12x^2y = 3x(a)$

6. Write the replacements for a and b in the following equations which will result in a true statement.

(a) $3x + 6y = 3(a + b)$

(b) $3xy + 2xz = x(a + b)$

7. Write the greatest common monomial that is a factor of each of the following pairs of monomials.

(a) $39a^2b$; $65ab^4$

(b) $24ab^2c^3$; $36a^3b^2c$

8. Rewrite each of the following polynomials as the product of its greatest monomial factor and another polynomial.

(a) $11t + 22t^2$

(b) $10s - 40rs$

(c) $11x^2 - 5xy$

(d) $5x^2y - 15x^3$

Criterion Test 03-07-01-3

1. Rewrite the following integers as the product of prime numbers.

(a) 72

(b) 24

2. Write all of the positive integral factors of each number.

(a) 19

(b) 20

3. Write the replacement for x which will make each statement true.

(a) $-17x = 17$

(b) $15x = -60$

4. Write the greatest common factor for each of the following pairs of integers.

(a) 315; 350

(b) 39; 65

5. Write a replacement for A which will result in a true statement in each of the following equations.

(a) $25x^4y^2 = -x^3(A)$

(b) $-15x^3y^2 = 3x^2y^2(A)$

6. Write the replacements for A and B in the following equations which will result in a true statement.

(a) $x^2 - 3x^2y = x^2(A + B)$

(b) $6x^2y - 3xy^2 = 3xy(A + B)$

7. Write the greatest common monomial that is a factor of each of the following pairs of monomials.

(a) $30r^2s$; $45rs$

(b) $132x^2yz^3$; $156x^3yz^2$

8. Rewrite each of the following polynomials as the product of its greatest monomial factor and another polynomial.

(a) $4rs - 12rt$

(b) $\pi r^2 + \pi s^2$

(c) $6ab - 12ac + 18ad$

(d) $2x^3 + 3x^2y + 2x$

ANSWERS

Criterion Test 03-07-01-1

1.a. $5 \cdot 17$
b. $2^3 \cdot 3^2$

2.a. {1,2,3,6,9,18}
b. {1,19}

3.a. 15
b. -1

4.a. 12
b. 35

5.a. x^2y
b. $-25xy^2$

6.a. a = x, b = 2y
b. a = 1, b = -3y

7.a. 13ab
b. 15rs

8.a. $5(x+y)$
b. $x(a-b)$
c. ~~$x(11x-5y)$~~ $2(3a^2+6ab+2b^2)$
d. ~~$5x^2(y-3x)$~~ $3ab(a+3b+4)$

Criterion Test 03-07-01-2

1.a. $5 \cdot 17$
b. $2^2 3^2$

2.a. {1,2,3,6,9,18}
b. {1,3,7,21}

3.a. 15
b. 5

4.a. 12
b. 15

5.a. x^2y
b. 4xy

6.a. a = x, b = 2y
b. a = 3y, b = 2z

7.a. 13ab
b. $12ab^2c$

8.a. $11t(1 + 2t)$
b. $10s(1 - 4r)$
c. $x(11x - 5y)$
d. $5x^2(y - 3x)$

Criterion Test 03-07-01-3

1.a. $2^2 \cdot 3^2$
b. $2^3 \cdot 3$

2.a. {1,19}
b. {1,2,4,5,10,20}

3.a. -1
b. -4

4.a. 35
b. 13

5.a. $-25xy^2$
b. -5x

6.a. a = 1, b = -3y
b. a = 2x, b = -y

7.a. 15rs
b. $12x^2yz^2$

8.a. $4r(s-3t)$
b. $\pi(r^2 + s^2)$
c. $6a(b-2c + 3d)$
d. $x(2x^2 + 3xy + 2)$

I.U. # 03-07-02

Expressing the product of the sum and difference of two monomials.

You will need to recall that:

In package # 03-06 you learned to find the product of two binomials.

Example: $(a + b)(x + y) = a(x + y) + b(x + y) = ax + ay + bx + by.$

OBJECTIVES:

1. Given five monomials, write the square of each.
2. Given two binomials which are the sum and difference of two monomials, write their product as a single binomial.

ACTIVITIES:

Study:

S & M: page 257, oral exercises 1-10 (Objective 1)

S & M: page 257, oral exercises 11-26 (Objective 2)

H, B & W: pages 322-323, ex. 1-23 odds (Objective 2)

Criterion Test # 03-07-02-1

1. Write the squares of the following monomials:

(a) $2s^3$

(b) $-2s^4$

(c) $3r^2s^3$

(d) $7x^3y^2$

(e) $6x$

2. Write the product of the following binomials:

(a) $(2a + 3b)(2a - 3b)$

(b) $(x - 2y)(x + 2y)$

Criterion Test # 03-07-02-2

1. Write the squares of the following monomials:

(a) $-6x^2y$

(b) $-8abc$

(c) $-m^2n^2$

(d) $-12xy^3z$

(e) $\frac{1}{2}x^2y^2$

2. Write the product of the following binomials:

(a) $(m + n)(m - n)$

(b) $(3r - s)(3r + s)$

Criterion Test # 03-07-02-3

1. Write the square of the following monomials:

(a) $2s^3$

(b) $-\frac{1}{2}x^2y^2$

(c) $6x$

(d) $-6x^2y$

(e) $-8abc$

2. Write the product of the following binomials:

(a) $(3ab - c)(3ab + c)$

(b) $(x - 2y)(x + 2y)$

Answers

Criterion test 03-07-02-1

1.a. $4s^6$

b. $4s^8$

c. $9r^4s^6$

d. $49x^6y^4$

e. $36x^2$

2.a. $4a^2 - 9b^2$

b. $x^2 - 4y^2$

Criterion test 03-07-02-2

1.a. $36x^4y^2$

b. $64a^2b^2c^2$

c. m^4n^4

d. $144x^2y^6z^2$

e. $1/4x^4y^4$

2.a. $m^2 - n^2$

b. $9r^2 - s^2$

Criterion test 03-07-02-3

1.a. $4s^6$

b. $1/4x^4y^4$

c. $36x^2$

d. $36x^4y^2$

e. $64a^2b^2c^2$

2.a. $9a^2b^2 - c^2$

b. $x^2 - 4y^2$

I.U. # 03-07-03

Factoring the difference of two squares.

03-07-03

OBJECTIVES:

1. Given a list of binomials, select those which are the difference of two squares.

2. Given a binomial that is the difference of two squares, write it as a product of two binomials. (five items)

ACTIVITIES:

Study:

S & M: page 258 (Objectives 1 and 2)

H, B & W: pages 328-329

Suggested Exercises:

S & M: page 259, oral exercises (Objective 1)

S & M: page 259, ex. 2-30 evens (Objective 2)

H, B & W: page 329, ex. 2-18 evens (Objective 2)

Criterion Test # 03-07-03-1

1. Select from the following binomials those which are the difference of two squares.

(a) $x^2 - y^2$

(b) $10r^2 - 27s^2$

(c) $16x - y^2$

(d) $4a^2 + 25b^2$

(e) $49 - z^2$

2. Write the following binomials in factored form:

(a) $x^2 - y^2$

(b) $x^2 - 16$

(c) $4x^2 - y^2$

(d) $25x^2 - 16a^2$

(e) $1 - 9n^2$

Criterion Test # 03-07-03-2

1. Select from the following binomials those which are the difference of two squares.

(a) $a^2 - b^2$

(b) $8r^2 - 9s^2$

(c) $25x^2 - 16y$

(d) $9a^2 + 16b^2$

(e) $36 - x^2$

2. Write the following binomials in factored form:

(a) $400 - a^2$

(b) $r^2 - 4s^2$

(c) $x^4 - y^2$

(d) $25x^2y^2 - 1$

(e) $49 - z^2$

Criterion Test # 03-07-3

1. Select from the following binomials those which are the difference of two squares.

(a) $x^4 - y^2$

(b) $20x^2y^2 - 1$

(c) $25x^2y - 1$

(d) $4a^2 + 1$

(e) $4a^2 - 1$

2. Write the following binomials in factored form:

(a) $16a^2b^2 - c^2$

(b) $100a^4b^4c^2 - 81$

(c) $1 - a^2z^2$

(d) $r^2 - s^2$

(e) $225a^2 - t^4$

Answers

Criterion test 03-07-03-1

1. a, e

2.a. $(x + y)(x - y)$
b. $(x + 4)(x - 4)$
c. $(2x + y)(2x - y)$
d. $(5x + 4a)(5x - 4a)$
e. $(1 + 3n)(1 - 3n)$

Criterion test 03-07-03-2

1. a, e

2.a. $(20 - a)(20 - a)$
b. $(r + 2s)(r - 2s)$
c. $(x^2 + y)(x^2 - y)$
d. $(5xy + 1)(5xy - 1)$
e. $(7 + z)(7 - z)$

Criterion test 03-07-03-3

1. a, e

2.a. $(4ab + c)(4ab - c)$
b. $(10a^2b^2c + 9)(10a^2b^2c - 9)$
c. $(1 + az)(1 - az)$
d. $(r + s)(r - s)$
e. $(15a + t^2)(15 - t^2)$

I.U. # 03-07-04

Squaring a binomial

OBJECTIVES

!. Given a binomial, write its square. (six items)

ACTIVITIES:

Study:

S & M: pages 260-1

H, B & W: page 320

Suggested Exercises:

S & M: page 261, ex. 1-23, odds

H, B & W: pages 320-1, ex. 3-27, multiples of three

Criterion Test 03-07-04-1

1. Square the following binomials

(a) $(4x - 7y)^2$

(b) $(5x + 11y)^2$

(c) $(2x^3 + y^2)^2$

(d) $(x^2y - 3)^2$

(e) $(7a^2 - 3b^2)^2$

(f) $(12 + x)^2$

Criterion Test # 03-07-04-2

1. Square the following binomials.

(a) $(2x + y)^2$

(b) $(m - n)^2$

(c) $(6x^2 - 5y)^2$

(d) $(2xy + 9m)^2$

(e) $(pq - zr)^2$

(f) $(3 - y)^2$

Criterion Test # 03-07-04-3

1. Square the following binomials.

(a) $(5a^2 + 4b)^2$

(b) $(2x - 3y^2)^2$

(c) $(b + ac)^2$

(d) $(xy - 10)^2$

(e) $(3m - 8n)^2$

(f) $(2a + 5b)^2$

Answers

Criterion test 03-07-04-1

1.a. $16x^2 - 56xy + 49y^2$

b. $25x + 110xy + 121y$

c. $4x^6 + 4x^3y^2 + y^4$

d. $x^4y^2 - 6x^2y + 9$

e. $49a^4 - 42a^2b^2 + 9b^4$

f. $144 + 24x + x^2$

Criterion test 03-07-04-2

1.a. $4x^2 + 4xy + y^2$

b. $m^2 - 2mn + n^2$

c. $36x^4 - 60x^2y + 25y^2$

d. $4x^2y^2 + 36xyn + 81m^2$

e. $p^2q^2 - 2pqzr + z^2r^2$

f. $9 - 6y + y^2$

Criterion test 03-07-04-3

1.a. $25a^4 + 40a^2b + 16b^2$

b. $4x^2 - 12xy^2 + 9y^4$

c. $b^2 + 2abc + a^2c^2$

d. $x^2y^2 - 20xy + 100$

e. $9m^2 - 48mn + 64n^2$

f. $4a^2 + 20ab + 25b^2$

I.U. 03-07-05

Factoring a trinomial square

OBJECTIVES:

1. Given a list of trinomials, select those which are perfect trinomial squares.

2. Given a trinomial square, write it as a binomial squared.

ACTIVITIES:

Study:

S & M: page 263 (Objectives 1 and 2)

H, B & W: pages 330-331 (Objective 1)

Suggested Exercises:

S & M: page 263, ex. 1-12 (Objective 1)

" page 264, oral ex. 13-23 odds (Objective 1)

" page 264, ex. 1-23 odds (Objective 2)

H, B & W: page 332, ex. 23-39 odds (Objective 2)

Criterion Test # 03-07-05-1

1. Indicate which of the following trinomials is a perfect square.

(a) $x^2 + 2x + 1$

(b) $x^2 - 10x + 25$

(c) $x^2 - 14x + 49$

(d) $2x^2 - 16x + 64$

(e) $9x^2 + 15x + 25$

(f) $x^2 + 18x - 81$

(g) $x^2 + 18x + 81$

2. Write the following trinomials as binomials squared.

(a) $x^2 + 4x + 4$

(b) $a^2 - 2ab + b^2$

(c) $4x^2 - 12xy + 9y^2$

Criterion Test # 03-07-05-2

1. Indicate which of the following trinomials is a perfect square.

(a) $x^2 + 8x + 16$

(b) $x^2 - 13x + 36$

(c) $x^2 - 32x + 64$

(d) $4x^2 + 20x + 100$

(e) $4x^2 - 6xy + 9y^2$

(f) $x^2 - 18x + 81$

(g) $4x^2 + 100x + 25a^2$

2. Write the following trinomials as binomials squared.

(a) $x^2 - 4x + 4$

(b) $9a^2 + 30ab + 25b^2$

(c) $x^2 + 12x + 36$

Criterion Test # 03-07-05-3

1. Indicate which of the following trinomials is a perfect square.

(a) $x^2 + 2x + 1$

(b) $x^2 - 13x + 36$

(c) $x^2 - 14x + 49$

(d) $4x^2 + 20x + 100$

(e) $9x^2 + 15x + 25$

(f) $x^2 - 18x + 81$

(g) $x^2 + 18x + 81$

2. Write the following trinomials as binomials squared.

(a) $x^2 + 6x + 9$

(b) $a^2 - 8a + 16$

(c) $81 + 36b + 4b^2$

Answers

Criterion test 03-07-05-1

1. a,b,c,f,g.

2.a. $(x + 2)^2$
b. $(a - b)^2$
c. $(2x - 3y)^2$

Criterion test 03-07-05-2

1. a,f

2.a. $(x - 2)^2$
b. $(3a + 5b)^2$
c. $(x + 6)^2$

Criterion test 03-07-05-3

1. a,c,f,g.

2.a. $(x + 3)^2$
b. $(a - 4)^2$
c. $(9 + 2b)^2$

I.U. " 03-07-06

Factoring the general trinomial

OBJECTIVES:

1. Given a trinomial of the form $x^2 + bx + c$, write it in factored form.
2. Given a trinomial of the form $x^2 - bx + c$, write it in factored form.
3. Given a trinomial of the form $x^2 - bx - c$, write it in factored form.
4. Given a trinomial of the form $x^2 + bx - c$, write it in factored form.
5. Given a trinomial of the form $ax^2 + bx + c$, write it in factored for
6. Given a trinomial of the form $ax^2 - bx + c$, write it in factored for
7. Given a trinomial of the form $ax^2 - bx - c$, write it in factored for
8. Given a trinomial of the form $ax^2 + bx - c$, write it in factored for
9. Given a factorable quadratic trinomial (not a trinomial square) which has no monomial factor, w ite it as the product of two binomials.

ACTIVITIES:

Study:

S & M: pages 267-8 (Objectives 1 and 2)

" page 270 (Objectives 3 and 4)

H, B & W: pages 333-4 (Objectives 1, 2, 3, 4)

S & M: pages 271-2 (Objectives 5, 6, 7, 8)

H, B & W: pages 335-6 (Objectives 5, 6, 7, 8)

Suggested Exercises:

S & M: page 269, ex. 1,2,3,4,9,10,17,18,20 (Objective 1)

" page 269, ex. 5,6,7,8,11,15,19 (Objective 2)

" page 271, ex. 3,4,5,14,15,23 (Objective 3)

" page 271, ex. 1,2,10,11,20 (Objective 4)

" page 272, ex. 1,2,3,4,21 (Objective 5)

" page 272, ex. 5,6,13 (Objective 6)

" page 272, ex. 8,9,11,15,20 (Objective 7)

" page 272, ex. 7,10,16,17 (Objective 8)

Criterion Test 03-07-06-1

1. Factor each of the following trinomials.

(a) $x^2 + 3x + 2$

(b) $x^2 - 3x + 2$

(c) $x^2 - 6x - 7$

(d) $x^2 + 12x - 13$

(e) $2x^2 + 5x + 2$

(f) $4x^2 - 8x + 3$

(g) $2x^2 - x - 1$

(h) $7x^2 + 2x - 5$

(i) $x^2 - 8x - 33$

(j) $a^2 - 12a + 36$

(k) $x^2 + 18x + 77$

(l) $6x^2 + 5xy - 21y^2$

Criterion Test # 03-07-06-2

1. Factor each of the following trinomials.

(a) $y^2 + 8y + 7$

(b) $x^2 - 6x + 5$

(c) $x^2 - 3x - 10$

(d) $x^2 + 6x - 16$

(e) $3x^2 + 5x + 2$

(f) $4x^2 - 9x + 2$

(g) $8b^2r^2 - 54br - 14$

(h) $2h^2 + h - 6$

(i) $x^6 + 2x^3y^3 + y^6$

(j) $10x^2 + 3xy - y^2$

(k) $6a^2 - ab - b^2$

(l) $x^2 + 6x - 40$

Criterion Test # 03-07-06-3

1. Factor each of the following trinomials.

(a) $x^2 + 9x + 14$

(b) $y^2 - 4y + 4$

(c) $x^2 - x - 6$

(d) $x^2 + 5x - 6$

(e) $3x^2 + 7x + 2$

(f) $12b^2 - 7b + 1$

(g) $18t^2 - 55t - 28$

(h) $6r^2 + 7r - 5$

(i) $14x^2 + 13x - 55$

(j) $a^2 - 12a + 36$

(k) $10x^2 + 3xy - y^2$

(l) $x^2 + 6x - 40$

ANSWERS

Criterion Test 03-07-06-1

1.a.	$(x + 2)(x + 1)$	g.	$(2x + 1)(x - 1)$
b.	$(x - 2)(x - 1)$	h.	$(7x - 5)(x + 1)$
c.	$(x - 7)(x + 1)$	i.	$(x - 11)(x + 3)$
d.	$(x + 13)(x - 1)$	j.	$(a - 6)(a - 6)$
e.	$(2x + 1)(x + 2)$	k.	$(x + 7)(x + 11)$
f.	$(2x - 1)(2x - 3)$	l.	$(3x + 7y)(2x - 3y)$

Criterion Test 03-07-06-2

1.a.	$(y + 7)(y + 1)$	g.	$(8br + 2)(br - 7)$
b.	$(x - 5)(x - 1)$	h.	$(2h - 3)(h + 2)$
c.	$(x - 5)(x + 2)$	i.	$(x^3 + y^3)(x^3 + y^3)$
d.	$(x + 8)(x - 2)$	j.	$(5x - y)(2x + y)$
e.	$(3x + 2)(x + 1)$	k.	$(3a + b)(2a - b)$
f.	$(x - 2)(4x - 1)$	l.	$(x + 10)(x - 4)$

Criterion Test 03-07-06-3

1.a.	$(x + 7)(x + 2)$	g.	$(9t + 4)(2t - 7)$
b.	$(y - 2)(y - 2)$	h.	$(3r + 5)(2r - 1)$
c.	$(x - 3)(x + 3)$	i.	$(7x + 11)(2x - 5)$
d.	$(x + 6)(x - 1)$	j.	$(a - 6)(a - 6)$
e.	$(3x + 1)(x + 2)$	k.	$(5x - y)(2x + y)$
f.	$(3b - 1)(4b - 1)$	l.	$(x + 10)(x - 4)$

I.U. # 03-07-07

Combining Several Types of Factoring

OBJECTIVES:

1. Be able to list, in order, the six steps used in factoring a polynomial product.

2. Given a polynomial, write it in factored form, if possible, by combining several types of factoring.

ACTIVITIES:

Study:

To factor a polynomial product:

1. Look for the greatest common monomial factor. Express in factored form.

2. Examine each factor. If one factor is a binomial, see if it is the difference of two squares. If it is, factor it.

3. Examine each factor. If it is a trinomial, see if it can be factored. If it can, factor it.

4. Repeat steps 2 and 3 until no further factoring is possible.

5. Write <u>all</u> of the factors.

6. Check by multiplication.

Example 1.

$2x^6y - 32x^2y$

$2x^2y(x^4 - 16)$ step 1

$2x^2y(x^2 + 4)(x^2 - 4)$ step 2

$2x^2y(x^2 + 4)(x + 2)(x - 2)$ steps 4 & 5 ← Answer

$2x^6y - 32x^2y$ step 6

Example 2:

$6y^2 - 28y - 480$

$2(3y^2 - 14y - 240)$ steps 1 & 5 ← Answer

$6y^2 - 28y - 480$ step 6

Example 3:

$90x^2 - 156x + 48$

$6(15x^2 - 26x + 8)$ step 1

$6(5x - 2)(3x - 4)$ steps 3 & 5 ← Answer

$90x^2 - 156x + 48$ step 6

Suggested Exercises:

S & M: Page 274, ex. 1-33 odds (Objective 2)

Criterion Test # 03-07-07-1

(Please cover Example 3 above before taking this test.)

1. List, in order, the six steps used in factoring a polynomial product.

2. Write the following polynomial products in factored form by combining several types of factoring.

(a) $5a^2 - 5$

(b) $2x^4 - 4x^2 + 2$

(c) $x^4 - 10x^2 + 9$

(d) $2x^4 - 8x^2$

(e) $a^3b - 4a^2b - 21ab$

Criterion Test # 03-07-07-2

1. List, in order, the six steps used in factoring a polynomial product.

2. Write the following polynomial products in factored form by combining several types of factoring.

(a) $m^4 - 1$

(b) $rx^8 - ry^8$

(c) $8\pi a^2 - 8\pi a - 10\pi$

(d) $x^3 - 36x$

(e) $6x^3 - 33x^2 + 15x$

Criterion Test # 03-07-07-3

1. List, in order, the six steps used in factoring a polynomial product.

2. Write the following polynomial products in factored form by combining several types of factoring.

(a) $x^4 - y^4$

(b) $6m + m^2 - m^3$

(c) $x^4 - 5x^2 + 4$

(d) $x^4 - 36x^2$

(e) $75 - 147x^2$

Answers

Criterion test 03-07-07-1

1. See Activities, page 41.

2.a. $5(a + 1)(a - 1)$
b. $2(x^2 - 1)^2$
c. $(x^2 - 9)(x^2 - 1)$
d. $2x^2(x + 2)(x - 2)$
e. $ab(a - 7)(a + 3)$

Criterion test 03-07-07-2

1. See Activities, page 41.

2.a. $(m^2 + 1)(m + 1)(m - 1)$
b. $r(x^4 + y^4)(x^2 + y^2)(x + y)(x - y)$
c. $2\pi(4a^2 - 4a - 5)$
d. $x(x + 6)(x - 6)$
e. $3x(2x - 1)(x - 5)$

Criterion test 03-07-07-3

1. See Activities, page 41.

2.a. $(x^2 + y^2)(x + y)(x - y)$
b. $m(2 + m)(3 - m)$
c. $(x + 2)(x - 2)(x + 1)(x - 1)$
d. $x^2(x + 6)(x - 6)$
e. $3(5 + 7x)(5 - 7x)$

I.U. # 03-07-08

Solving Polynomial Equations by Factoring

OBJECTIVES:

1. When asked to state the zero-product property of real numbers you will write, "For all real numbers a and b, ab = 0 if and only if a = 0 or b = 0.

2. Given a polynomial equation, write it in standard form.

3. Be able to list, in order, the four steps used in solving a polynomial equation by factoring.

4. Given a polynomial equation with rational roots, write a step-by-step solution by using factoring.

ACTIVITIES:

Study:

S & M: Page 275 (Objective 1)

To solve a polynomial equation:

1. Write in standard form.

2. Factor left-hand member.

3. Set each factor equal to zero and solve the resulting equations.

4. Check in the original equation.

S & M: Pages 277-279 (Objectives 2,3,4)

Suggested Exercises:

S & M: Page 276, ex. 1-8 (application of objective 1)

Page 279, ex. 1-31 odds (objectives 2 and 3)

Criterion Test # 03-07-08-1

1. State the zero-product property of real numbers.

2. Write the following polynomial equations in standard form.

(a) $9x^2 = 16$

(b) $x^2 = 9x - 14$

(c) $x^2 = 24 + 5x$

3. List, in order, the four steps used in solving a polynomial equation by factoring.

4. Write a step-by-step solution of the following equations by using factoring.

(a) $x^2 + 6x + 5 = 0$

(b) $x^2 + 7x = -6$

(c) $x^2 - 5x = 0$

(d) $4x^2 = 9$

(e) $x^2 = -6x - 9$

Criterion Test # 03-07-08 2

1. State the zero-product property of real numbers.

2. Write the following polynomial equation in standard form.

(a) $5y = 36 - y^2$

(b) $3y = 28 - y^2$

(c) $3x = x^2$

3. List, in order, the four steps used in solving a polynomial equation by factoring.

4. Write a step-by-step solution of the following equation by using factoring.

(a) $3t^2 + t = 2$

(b) $6y^2 - 4y - 10 = 0$

(c) $5x^2 = 6x$

(d) $3x^2 = 7x + 20$

(e) $0 = 2y^2 + 12y + 10$

Criterion Test # 03-07-08-3

1. State the zero-product property of real numbers.

2. Write the following polynomial equations in standard form.

(a) $x^2 = 24 + 5x$

(b) $3y = 28 - y^2$

(c) $9x^2 = 16$

3. List, in order, the four steps used in solving a polynomial equation by factoring.

4. Write a step-by-step solution of the following quadratic equations by using factoring.

(a) $-5x = -12x^2 + 2$

(b) $14 = x(x - 5)$

(c) $0 = -3x + 5x^2$

(d) $x^2 - 5x = 18 + 2x$

(e) $25x^2 - 16 = 0$

Answers

Criterion test 03-07-08-1

1. See objective 1, page 46

2.a. $9x^2 - 16 = 0$
b. $x^2 - 9x + 14 = 0$
c. $x^2 - 5x - 24 = 0$

3. See Activities page 53.

4.a. $x^2 + 6x + 5 = 0$
$(x + 5)(x + 1) = 0$
$x + 5 = 0$ or $x + 1 = 0$
$x = -5$ or $x = -1$

b. $x^2 + 7x = -6$
$x^2 + 7x + 6 = 0$
$(x + 6)(x + 1) = 0$
$x + 6 = 0$ or $x + 1 = 0$
$x = -6$ or $x = -1$

c. $x^2 - 5x = 0$
$x(x - 5) = 0$
$x = 0$ or $x5 = 0$
$x = 0$ or $x = 5$

d. $4x^2 = 9$
$4x^2 - 9 = 0$
$(2x + 3)(2x - 3) = 0$
$2x + 3 = 0$ or $2x - 3 = 0$
$2x = -3$ or $2x = 3$
$x = -3/2$ or $x = 3/2$

e. $x^2 = -6x - 9$
$x^2 + 6x + 9 = 0$
$(x + 3)^2 = 0$
$x + 3 = 0$
$x = -3$

Criterion test 03-07-08-2

1. See objective 1, page 46.

2a. $y^2 + 5y - 36 = 0$
b. $y^2 + 3y - 28 = 0$
c. $x^2 - 3x = 0$

3. See Activities page 53.

4a. $3t^2 + t = z$
$3t^2 + t - z = 0$
$(3t - 2)(t + 1) = 0$
$3t - z = 0$ or $t + 1 = 0$
$3t = 2$ or $t = -1$
$t = 2/3$ or $t = -1$

b. $6y^2 - 4y - 10 = 0$
$2(3y^2 - 2y - 5) = 0$
$2(3y - 5)(y + 1) = 0$
$3y - 5 = 0$ or $y + 1 = 0$
$3y = 5$ or $y = -1$
$y = 5/3$ or $y = -1$

c. $5x^2 = 6x$
$5x^2 - 6x = 0$
$x(5x - 6) = 0$
$x = 0$ or $5x - 6 = 0$
$x = 0$ or $5x = 6$
$x = 0$ or $x = 6/5$

d. $3x^2 = 7x + 20$
$3x^2 - 7x - 20 = 0$
$(3x + 5)(x - 4) = 0$
$3x + 5 = 0$ or $x - 4 = 0$
$3x = -5$ or $x = 4$
$x = -5/3$ or $x = 4$

e. $0 = 2y^2 + 12y + 10$
$0 = 2(y^2 + 6y + 5)$
$0 = 2(y + 5)(y + 1)$
$y + 5 = 0$ or $y + 1 = 0$
$y = -5$ or $y = -1$

Criterion test 03-07-08-3

1. See objective 1, page 46.

2.a. $x^2 - 5x - 24 = 0$
b. $y^2 + 3y - 28 = 0$
c. $9x^2 - 16 = 0$

3. See activities page 53.

4.a. $-5x = -12x^2 + 2$
$12x^2 - 5x - 2 = 0$
$(4x + 1)(3x - 2) = 0$
$4x + 1 = 0$ or $3x - 2 = 0$
$4x = -1$ or $3x = 2$
$x = -1/4$ or $x = 2/3$

b. $14 = x(x - 5)$
$14 = x^2 - 5x$
$0 = x^2 - 5x - 14$
$0 = (x - 7)(x + 2)$
$x - 7 = 0$ or $x + 2 = 0$
$x = 7$ or $x = -2$

c. $0 = -3x + 5x^2$
$0 = x(-3 + 5x)$
$x = 0$ or $-3 + 5x = 0$
$x = 0$ or $-3 = -5x$
$x = 0$ or $x = 3/5$

d. $x^2 - 5x = 18 + 2x$
$x^2 - 7x - 18 = 0$
$(x - 9)(x + 2) = 0$
$x - 9 = 0$ or $x + 2 = 0$
$x = 9$ or $x = -2$

e. $25x^2 - 16 = 0$
$(5x - 4)(5x + 4) = 0$
$5x - 4 = 0$ or $5x + 4 = 0$
$5x = 4$ or $5x = -4$
$x = 4/5$ or $x = -4/5$

I.U. " 03-07-09

Solving

Word Problems

You will need to recall these formulas:

Area of a triangle: $A = \frac{1}{2}bh$; b = base, h = altitude

Area of a rectangle: $A = bh$; b = base, h = altitude

Perimeter of a triangle: $P = a + b + c$; a,b,c = lengths of sides

Perimeter of a rectangle: $P = 2L + 2w$; L = length, w = width

Pythagorean relationship (right triangle): $a^2 + b^2 = c^2$;
a and b = legs, c = hypotenuse

Distance a free falling object travels: $d = rt + 16t^2$; r = initial velocity in feet per second, t = time in seconds.

Height reached by an object projected upward: $h = rt - 16t^2$;
r = starting speed in feet per second, t = time in seconds.

You will not be expected to memorize the last two formulas.

OBJECTIVES:

1. Write, from memory, the formulas for: (1) the area of a triangle, (2) the area of a rectangle, (3) the perimeter of a triangle, (4) the perimeter of a rectangle, and (5) Pythagorean relationship

2. Write in order the seven steps followed in solving a word problem.

3. Given a word problem which leads to a polynomial equation, write that equation.

4. Given a problem which leads to a polynomial equation, write a solution to the problem.

ACTIVITIES:

Study:

To solve a word problem:

1. Read and understand the problem.
2. Express all unknown quantities in terms of a variable.
3. Write an equation.
4. Solve the equation.
5. Reject unreasonable roots.
6. Write the answer.
7. Check.

This package: formulas in "you will need to recall", page 52 (Objective 1)

S & M: pages 280-1 (Objectives 2 and 3)

H, B & W: page 375, any exercises except #5.

Criterion Test 03-07-09-1

1. Write the following formulae:

(a) The area of a triangle.

(b) The area of a rectangle.

(c) The perimeter of a triangle.

(d) The perimeter of a rectangle.

(e) The Pythagorean relationship.

2. Write, in order, the seven steps followed in solving a word problem.

3. Write an equation for each of the following problems, and use it to find the solution to the problem.

(a) A flower bed is twice as long as it is wide and its area is 162 sq. ft. How wide is it?

(b) The square of a certain number is 9 more than 8 times the number. Find the number.

(c) The product of two consecutive odd integers is 323. Find the numbers.

(d) Joe Blow is both a baseball fan and a mathematician. He discovered one day that Art Jenkins had hit the same number of singles as doubles; and that their product, increased by 36, was 232. How many singles had he hit?

Criterion Test 03-07-09-2

1. Write the following formulae:

(a) The area of a triangle.

(b) The area of a rectangle.

(c) The perimeter of a triangle.

(d) The perimeter of a rectangle.

(e) The Pythagorean relationship.

2. Write, in order, the seven steps followed in solving a word problem.

3. Write an equation for each of the following problems, and use it to find the answer to the problem.

(a) The width of a desk is 26 in. less than its length and its area is 1200 sq. in. How wide is the desk?

(b) The sum of a number and its square is 90. Find the number.

(c) Jake has a certain number of nickels. The square of their total value is 400 cents. How many nickels does he have?

(d) The sum of the squares of two consecutive negative odd integers is 74. Find the integers.

Criterion Test 03-07-09-3

1. Write the following formulae:

(a) The area of a triangle.

(b) The area of a rectangle.

(c) The perimeter of a triangle.

(d) The perimeter of a rectangle.

(e) The Pythagorean relationship.

2. Write, in order, the seven steps followed in solving a word problem.

3. Write an equation for each of the following problems, and use it to find the answer to the problem.

(a) The number of inches in the perimeter of a certain square is the same as the number of square inches in its area. Find the measure of a side of the square.

(b) The product of two consecutive integers is 552. Find the numbers.

(c) In a certain auditorium the number of rows is 8 less than the number of seats per row. There are 1428 seats in all. How many seats are there in each row?

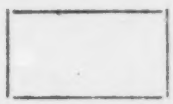
(d) Mary had two types of coins. To be exact, she has ten more of one type than of the other. The product of the number of one type and the number of the other type happens to equal the total number of cents that the coins are worth: 200 cents. How many coins of each kind has she?

Answer

Criterion test 03-07-09-1

1. See page 53

2. See page 54

3a.  x x(2x) = 162 9 ft. wide

2x

b. $x^2 = 8x + 9$ 9 or -1
c. $x(x + 2) = 323$ 17 and 19 or -17 and -19
d. $x^2 + 36 = 232$ 14 singles

Criterion test 03-07-09-2

1. See page 53
2. See page 54

3a. x - 26 $x(x - 26) = 1200$ 24 in. wide
($x = 50$)

x

b. $x + x^2 = 90$ -10 or +9
c. $(5x)^2 = 400$ 4 nickels.
d. $x^2 + (x + 2)^2 = 74$ -7 and -5

Criterion test 03-07-09-3

1. See page 53
2. See page 54

3a. x $2x + 2x = x^2$ 4 inches

x

b. $x(x + 1) = 552$ -24 and -23 or 23 and 24
c. $x(x - 8) = 1428$ 42 seats
d. $x(x + 10) = 200$ 10 and 20